5th GRACM International Congress on Computational Mechanics
Limassol, 29 June – 1 July, 2005

WAVE DISPERSION IN DRY SAND BY EXPERIMENTAL, ANALYTICAL AND NUMERICAL METHODS

T. Wichtmann[a], E. I. Sellountos[b], S. V. Tsinopoulos[c], A. Niemunis[a], S. Papargyri-Beskou[d], D. Polyzos[b], T. Triantafyllidis[a] and D. E. Beskos[c]

[a]Institute of Soil Mechanics and Foundation Engineering
Ruhr-University Bochum
D-44801 Bochum, Germany

[b]Department of Mechanical and Aeronautical Engineering
University of Patras
GR-26500 Patras, Greece

[c]Department of Civil Engineering
University of Patras
GR-26500 Patras, Greece
e-mail: d.e.beskos@upatras.gr (D. E. Beskos)

[d]Mechanics Division, General Department, School of Technology
Aristotle University of Thessaloniki
GR-54006 Thessaloniki, Greece

Keywords: Wave dispersion, Granular medium, Dry sand, Experimental method, Gradient elasticity, Porous elasticity, Boundary element method.

Abstract. *The dispersion of longitudinal and shear elastic waves propagating in dry sand cylindrical specimens is studied by experimental, analytical and numerical methods. Wave propagation in the specimen is studied experimentally with the aid of a triaxial cell system equipped with pairs of piezoelectric ceramic elements for the generation and reception of longitudinal and shear waves. Two different models taking into account the microstructural properties of dry sand, a gradient elastic and an elastic porous model, are also employed to study wave dispersion analytically. The same problem is also studied by the boundary element method in the frequency domain in conjunction with the iterative effective medium approximation on the assumption that dry sand is a porous medium with equally sized spherical pores. The results of all methods are critically compared and discussed.*

1 INTRODUCTION

Wave propagation in granular materials, such as sands, sandstones, grains, ceramics, porous bones and pressed powders is an important field of study in geotechnical engineering, geophysics, bioengineering and material science and engineering. Wave propagation studies in these materials aim at the understanding of their internal microstructure, determining their mechanical properties and establishing accurate and efficient methods for the evaluation of their response to dynamic loading.

Among the many published works on elastic wave propagation in granular media, one can mention experimental studies[1-5], analytical studies[6-13] and numerical studies[14-17]. To the authors' best knowledge, with the exception of references[4, 15] where there is a comparison between experimental and analytical results, no comparisons between the results of the above three kinds of methods have been reported in the recent literature.

The present work represents a moderate effort towards a comparison of experimental, analytical and numerical methods as applied to the study of wave propagation in granular media. More specifically, this work studies dispersion of longitudinal (P) and shear (S) waves in dry sand assuming small-amplitude harmonic waves. Experimental dispersion curves are obtained with the aid of a triaxial cell system with pairs of piezoelectric transducers for the generation and reception of waves at the bottom and top, respectively, of a cylindrical dry sand specimen pre-pressured uniformly. The dispersion curves of the specimen are also obtained on the basis of two different microstructural elasticity theories: the linear elasticity with pores (voids) due to Cowin and Nunziato[7] and the linear dipolar gradient elasticity due to Mindlin[18]. Finally, dispersion is also studied by a numerical method that combines the iterative effective medium approximation technique[14, 15] used for the elastic porous medium of the sand specimen with the frequency domain boundary element method used for the wave scattering analysis. All the methods are in agreement for the cases of high frequencies for which there are essentially no dispersional waves. However, for the cases of low frequencies there is disagreement between the methods, which is clearly pointed out.

2 EXPERIMENTAL METHOD

The phase velocities of longitudinal (P) and shear (S) waves propagating in dry sand cylindrical specimens have been measured with the aid of a triaxial cell test device instrumented with pairs of piezoelectric elements. These elements (three types of them) are placed at the specimen end plates and serve to generate and receive P and S waves. The experimental technique for measuring wave velocities is shown schematically in Fig. 1. A single sine voltage is generated by a function genarator, then is amplified and finally is conducted towards an oscilloscope and towards the selected piezoelectric transducer placed at the lower specimen end plate. The electrical impulse causes a deformation of the piezoelectric element and thus the emission of a wave. Conversely, the wave arrival at the upper specimen end plate has as a result the generation of an eclectric output to the corresponding receiver. Finally, the received signal after amplification is transferred to a computer for post-processing purposes. The wave velocity is then calculated as the quotient of the travel length (length of the specimen) and travel time (time difference between the start of the transmitted signal and the first arrival of the received signal). Since every generated sine signal is associated with a specific frequency, every measured wave velocity becomes a function of frequency, thereby creating a phase velocity versus frequency relation, i.e., a dispersion curve.

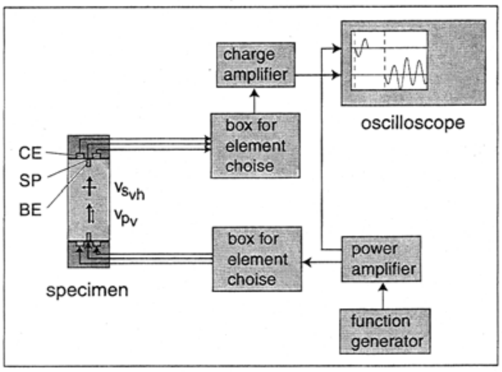

Figure 1. Scheme of the test device

In the present work, cylindrical triaxial specimens 10cm in diameter and 5cm in height were used. The small specimen height was chosen in order to reduce the influence of material damping and the reflections of the waves at the specimen boundaries. A medium coarse dry sand with mean diameter $d = 0.55$mm and maximum and minimum void ratios $e_{\max} = 0.874$ and $e_{\min} = 0.577$, respectively, was used. The specimens were prepared by pluviating dry sand out of a funnel through air into half cylinder moulds. Having placed the specimen top cap, sealed the membrane and applied a vacuum of 50 kPa to the grain skeleton, the half-cylinder moulds were removed. The geometry of the specimen was measured and the plexiglass cylinder of the pressure cell was mounted. The vacuum in the grain skeleton was gradually replaced by the pressure in the cell keeping the effective isotropic stress $\sigma_1 = \sigma_3$=50 kPa, where the indices 1 and 3 stand for the axial and the radial component, respectively.

The experimental measurements were performed on sand specimens having three different void ratio values (e = 0.797, 0.667, 0.571) under four effective isotropic stress values $\left(\sigma_1 = \sigma_3 = 50, 100, 200, 400 \text{ kPa}\right)$ in order to study the influence of the relative density of the soil as well as the surrounding pressure on the wave propagation velocities. The frequencies of the transmitted waves were varied from 5 kHz up to 200 kHz in steps of 5 kHz.

3 ANALYTICAL METHODS

According to the theory of linear elastic materials with voids due to Cowin and Nunziato[7], the void volume in elastic porous materials changes with the deformation. Thus, the governing equations of motion for such materials has the form

$$\mu u_{i,jj} + (\lambda + \mu) u_{j,ji} + \beta \phi_{,i} = \rho \ddot{u}_i \tag{1}$$

$$\alpha\,\phi_{,ii} - \gamma\,\dot{\phi} - \xi\,\phi - \beta\,u_{\kappa,\kappa} = \rho\,\delta\,\ddot{\phi} \tag{2}$$

where u_i is the displacement vector, ϕ the change in void volume fraction, ρ the mass density, λ and μ the Lamè constants, α,β,γ,ξ and δ material constants, indicial notation holds, commas indicate spatial differentiation and overdots differentiation with respect to time t. Assuming harmonic waves of the form

$$u_i\left(x_i,t\right)=\overline{u}\,d_i\,e^{-i\left[(\omega/V)m_i\,x_i+\omega\,t\right]},\qquad \phi\left(x_i,t\right)=\overline{\phi}\,e^{-i\left[(\omega/V)m_i\,x_i+\omega\,t\right]} \tag{3}$$

where d_i and m_i are unit vectors indicating the directions of displacement and propagation, respectively, V the phase velocity, ω the circular frequency and $\overline{u}$ and $\overline{\phi}$ amplitudes, one can prove that, while shear waves propagate with $V_s = c_s = \sqrt{\mu/\rho}$ (no dispersion), longitudinal waves propagate with a velocity V_p, which depends on frequency $\omega = 2\pi f$, where f is the frequency in Hz. This V_p versus f dispersion relation for the special case of $\xi = \gamma = 0$ takes the form

$$V_p^2 = \left\{\left[\alpha/\delta+\left(\lambda+2\mu\right)\right]\pm\sqrt{\left[\alpha/\delta-\left(\lambda+2\mu\right)\right]^2+\beta^2/\pi^2\delta f^2}\right\}/2\rho \tag{4}$$

According to the dipolar gradient theory of elasticity due to Mindlin[18], as simplified in [13, 4, 16], one can have the governing equation of motion of an elastic body with microstructure in the form

$$\mu\,u_{i,jj} + \left(\lambda+\mu\right)u_{j,ji} + g^2\left(\mu\,u_{i,jj} + \left(\lambda+\mu\right)u_{j,ji}\right)_{,\kappa\kappa} = \rho\,\ddot{u}_i - \rho\,h^2\ddot{u}_{,\kappa\kappa} \tag{5}$$

where g and h are the gradient coefficients of volumetric strain energy and velocity, respectively.

Assuming harmonic waves of the form

$$\begin{aligned}\mathbf{u} &= \nabla\phi + \nabla\times\mathbf{A}\\ \nabla\phi &= \mathbf{m}\,e^{-i\left[(\omega/c_p)\mathbf{m}\cdot\mathbf{x}+\omega t\right]}\\ \nabla\times\mathbf{A} &= \mathbf{d}\,e^{-i\left[(\omega/c_s)\mathbf{m}\cdot\mathbf{x}+\omega t\right]}\end{aligned} \tag{6}$$

where $c_p^2 = \left(\lambda+2\mu\right)/\rho$ and $c_s^2 = \mu/\rho$ stand for the longitudinal and shear wave velocities of classical wave propagation, one can prove that both longitudinal and shear waves are here dispersive and the $V_{p,s}$ versus $f = \omega/2\pi$ relation takes the form

$$V_{p,s}^2 = 8\pi^2 c_{p,s}\,g^2 f^2 \Big/ \left[-\left(c_{p,s}^2 - 4\pi^2h^2f^2\right)+\sqrt{\left(c_{p,s}^2-4\pi^2h^2f^2\right)^2+16\pi^2c_{p,s}^2\,g^2f^2}\right] \tag{7}$$

4 NUMERICAL METHOD

The numerical method used here to study wave dispersion in dry sand is based on the iterative effective medium approximation [14, 15] applied with the aid of an advanced frequency domain boundary element method for axisymmetric problems [19].

An elastic wave propagating in a soil medium, which is strongly inhomogeneous, can be considered as the sum of a mean wave and a number of fluctuating waves. The mean wave exists in a homogeneous effective medium with equivalent properties, while the fluctuating waves are the result of the multiple scattering of the mean wave by the randomly distributed material variations with respect to those of the effective medium. Under this consideration, the average of fluctuating fields should be vanishing at any direction within the effective medium. This self-consistent condition, for the case of a material consisting of grains and voids (dry sand) can take the simplified form [14, 15]

$$v_g g^{(1)}\left(\hat{\mathbf{k}},\hat{\mathbf{k}}\right) + \left(1-v_g\right)g^{(2)}\left(\hat{\mathbf{k}},\hat{\mathbf{k}}\right) = 0 \tag{8}$$

where v_g represents the volume fraction of the sand grains and $g^{(1)}(\hat{\mathbf{k}},\hat{\mathbf{k}})$ and $g^{(2)}(\hat{\mathbf{k}},\hat{\mathbf{k}})$ are the forward scattering amplitudes taken by the solution of the two single wave scattering problems illustrated in Fig. 2.

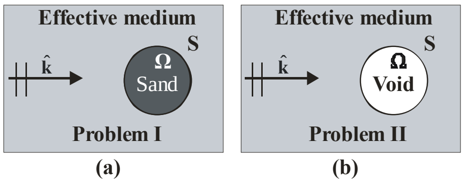

Figure 2: Single wave scattering problems of a mean wave propagating in the effective medium; a) the scatterer is the sand grain (problem I) and b) the scatterer is a void inclusion with identical to the sand grain geometrical properties (problem II).

In the present work, the self-consistent condition (8) is satisfied numerically by following an iterative procedure. This iterative effective medium approximation (IEMA) procedure can be summarized as follows: Consider a soil medium consisting of identical spherical sand grains with density, shear and bulk moduli, ρ_g, μ_g and K, respectively in a volume fraction equal to v_g. A harmonic elastic wave either longitudinal (P) or shear (S) is propagated through the soil. Due to the inhomogeneity, multiple scattering occurs which makes the mean wave both dispersive and attenuated. Thus, its complex wavenumber $k_d^{eff}(\omega)$ can be written as

$$k_d^{eff}(\omega) = \frac{\omega}{V_d^{eff}(\omega)} + i\alpha_d^{eff}(\omega) \tag{9}$$

where $V_d^{eff}(\omega)$ and $\alpha_d^{eff}(\omega)$ are the frequency dependent phase velocity and attenuation coefficient, respectively, of the mean plane wave, while the subscript d denotes either longitudinal $(d \equiv \mathrm{P})$ or shear $(d \equiv \mathrm{S})$ waves. Next, the soil medium is replaced by an elastic homogeneous and isotropic medium with effective shear and bulk moduli μ^{eff} and K^{eff}, respectively, given by the formulas of Mac Kenzie[20]:

$$\begin{aligned} K^{eff} &= v_g K_g \left[1 - \frac{3}{4}(1 - v_g)\frac{K_g}{\mu_g} \right] \\ \mu^{eff} &= \mu_g \left[1 - 5(1 - v_g)\frac{3K_g + 4\mu_g}{9K_g + 8\mu_g} \right] \end{aligned} \tag{10}$$

At the first step of the IEMA method, the effective density is assumed to be

$$\left(\rho^{eff}\right)_{step1} = v_g\, \rho_g \tag{11}$$

Then, the real effective wave number $(k_d^{eff})_{step1}$ is straightforwardly evaluated through (9), using the material properties (10) and (11). At the second step of the IEMA, by utilizing the material properties obtained from the first step, the two single wave scattering problems, illustrated in Fig. 1, are solved. The solution of the scattering problems is accomplished numerically by means of an advanced 3-D axisymmetric boundary element code [19]. Combining relation (8) with a dispersion relation due to Foldy [21], one can arrive at the new effective wave number of the mean wave, given by

$$\left[\left(k_d^{eff}\right)_{step2}\right]^2 = \left[\left(k_d^{eff}\right)_{step1}\right]^2 + \frac{3v_g}{r^3}\, g_d\left(\hat{\mathbf{k}}, \hat{\mathbf{k}}\right) \tag{12}$$

where r is the radius of the sand grain. The new, complex now, density $(\rho^{eff})_{step2}$ is evaluated from the $(k_d^{eff})_{step2}$ and relations (10). Then the second step is repeated with the material properties (10) and the new density $(\rho^{eff})_{step2}$ until the self consistent condition (8) is satisfied. Finally, from the relation (9), the frequency dependent effective phase velocity $V_d^{eff}(\omega)$ and the attenuation coefficient $\alpha_d^{eff}(\omega)$, are computed.

5 COMPARISON OF RESULTS

Due to space limitations only a few results are presented herein. They correspond to the case of dry sand with grains of mean radius r = 0.275 mm, volume fraction V_g=0.6 or void ratio e = 0.667, Lame' constants λ_g = 3050

MPa and μ_g = 558 MPa and mass density $\rho_g = 2633\,\text{kg/m}^3$. On the basis of Eqs (10) one can obtain $K^{eff} = 211\,\text{MPa}$, $\mu^{eff} = 137\,\text{MPa}$ and hence $\lambda^{eff} = 120\,\text{MPa}$ and $\rho^{eff} = \rho_g V_g = 1580\,\text{kg/m}^3$. These effective properties have been used in connection with the two analytical and the numerical method reported herein. Figures 3 and 4 show the dispersion curves for P and S waves, respectively, as obtained by the four methods considered in this work, i.e., the experimental, the two analytical and the numerical method. The analytical method of porous elasticity [7] was used with material coefficient ratios $\alpha/\delta = 24\times10^7\,\text{N/m}^2$ and $\beta^2/\delta = 92\times10^{26}\,\text{N/m}^4\text{s}^2$, while that one of gradient elasticity[18] with material constants $g = 55\times10^{-5}\text{m}$ and $h = g/1.034\,\text{m}$ as well as $g = 16.5\times10^{-5}\text{m}$ and $h = \text{g}/1.030\,\text{m}$.

One can observe that the experimental results for both types of waves (P and S) show essentially no dispersion. The gradient elasticity[18] results are close to the experimental ones and indicate a very small dispersion, which however, shows a slight increase for high frequencies. On the other hand, the analytical method of porous elasticity[7] for P waves shows a rapid increase of phase velocity with decreasing values of frequencies in the low frequency range. However, it shows a good agreement with the experimental results for higher frequencies. This theory shows zero dispersion for S waves. Finally, the results of the numerical method of IEMA/BEM[14,15] are rather close with the experimental ones only for low frequencies and exhibit more dispersion than any other of the methods considered in this work. It appears that the gradient elasticity method approaches experimental results better than the other methods and exhibits very small dispersion due to its internal microstructure. However, a slight dispersion increase for high frequencies is noticeable. Experimental results show zero dispersion. However, measurements near very small frequencies $(f \le 5\,\text{kHz})$ cannot be considered as reliable, leaving a question whether or not there is really dispersion in that range.

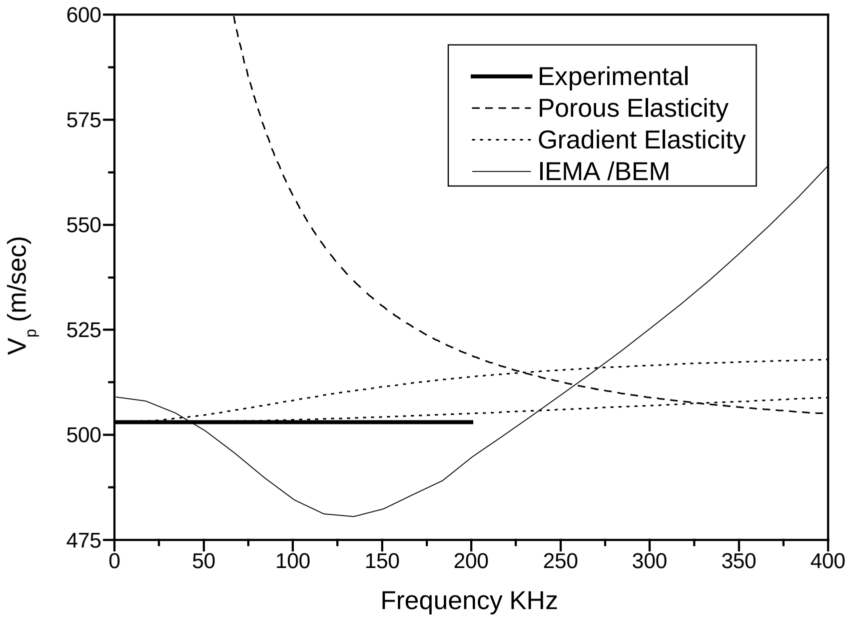

Figure 3. Dispersion curves for P waves

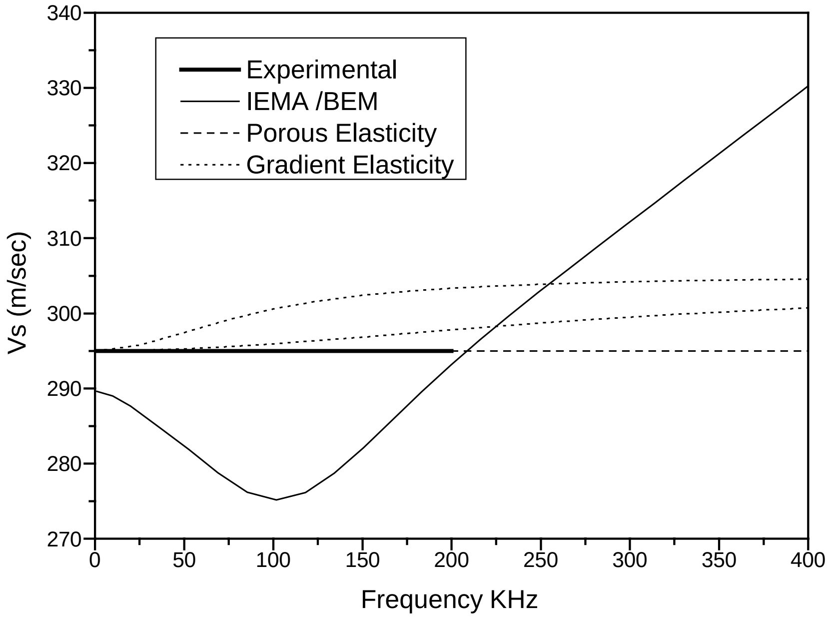

Figure 4. Dispersion curves for S waves

6 CONCLUSIONS

On the basis of the preceding analysis and discussion the following conclusions can be stated:

1) Experimental results exhibit zero dispersion for all frequencies, eventhough measurements for very low frequencies cannot be considered as very reliable.
2) The analytical method of dipolar gradient elasticity due to Mindlin [18] shows results very close to the experimental results, eventhough the values of velocities are always above the experimental ones.
3) The analytical method of porous elasticity due to Cowin and Nunziato [7] agrees with the experimental results only for high frequencies and exhibits values of velocity approaching infinity for low frequencies.
4) The numerical method of IEMA/BEM[14,15] provides results rather close to the experimental ones only for low frequencies and indicates more dispersion then any of the other methods.
5) It is apparent that more in depth investigations are needed in order to more clearly understand the dynamic behavior of dry sand and assess the performance of the various analytical and numerical models in connection not only with wave dispersion, but in addition, with wave attenuation and the solution of wave propagation boundary value problems in dry sand.

ACKNOWLEDGEMENTS

The authors acknowledge with thanks the support provided to them through the IKYDA-DAAD programs in the framework of the Greek – German Scientific Co-operation. Thanks are also due to Miss M. Dimitriadi for her excellent typing of the manuscript.

REFERENCES

[1] Bathia, S., Blaney, G., Editors (1991), “Recent Advances in Instrumentation, Data Acquisition and Testing in Soil Dynamics”, ASCE Geotechnical Special Publication No 29, ASCE, New York.

[2] Brignoli, E. G. M., Gotti, M., Stokoe, K. H. II (1996), “Measurement of shear waves in laboratory specimens by means of piezoelectric transducers”, Geotechnical Testing Journal 19, pp. 384-397.

[3] Wichtmann, T., Triantafyllidis, T. (2004), “Influence of a cyclic and dynamic loading history on dynamic properties of dry sand, Part II: cyclic axial preloading”, Soil Dynamics and Earthquake Engineering 24, pp. 789-803.

[4] Aggelis, D. G., Philippidis, T. P., Tsinopoulos, S. V., Polyzos, D. (2004), “Wave dispersion in concrete due to microstructure”, *CD-ROM Proceedings of the 2004 Int. Conference on Computational & Experimental Engineering & Sciences,* Madeira, Portugal, 26-29 July.

[5] Donald, J. A., Butt, S. D. (2005), Experimental technique for measuring phase velocities during triaxial compression tests", International Journal of Rock Mechanics and Mining Sciences 42, pp. 307-314.
[6] Nunziato, J. W., Walsh, E. K. (1977), "Small-amplitude wave behavior in one-dimensional granular solids", Journal of Applied Mechanics (ASME) 44, pp. 559-564.
[7] Cowin, S. C., Nunziato, J. W. (1983), "Linear elastic materials with voids", Journal of Elasticity 13, pp. 125-147.
[8] Walton, K. (1988), "Wave propagation within random packing of spheres", Geophysical Journal 92, pp. 89-97.
[9] Chang, C. S., Gao, J. (1995), "Non-linear dispersion of plane wave in granular media", International Journal of Non-Linear Mechanics 30, pp. 111-128.
[10] Chang, C. S., Gao, J. (1997), "Wave propagation in granular rod using high-gradient theory", Journal of Engineering Mechanics (ASCE) 123, pp. 52-59.
[11] Tsepoura, K. G., Papargyri-Beskou, S., Polyzos, D., Beskos, D. E. (2002), "Static and dynamic analysis of gradient elastic bars in tension", Archive of Applied Mechanics 72, pp. 483-497.
[12] Papargyri-Beskou, S., Polyzos, D., Beskos, D. E. (2003), "Dynamic analysis of gradient elastic flexural beams", Structural Engineering and Mechanics 15, pp. 705-716.
[13] Georgiadis, H. G., Vardoulakis, I., Velgaki, E. G. (2004), "Dispersive Rayleigh-wave propagation in microstructured solids characterized by dipolar gradient elasticity", Journal of Elasticity 74, pp. 17-45.
[14] Verbis, J. T., Tsinopoulos, S. V., Polyzos, D. (2002), "Elastic wave propagation in fiber reinforced composite materials with non-uniform distribution of fibers", Computer Modeling in Engineering and Sciences (CMES) 3, pp. 803-814.
[15] Aggelis, D., Tsinopoulos, S. V., Polyzos, D. (2004), "A practical IEMA for predicting wave dispersion in particulate composites and suspensions", Journal of the Acoustical Society of America 116, pp. 3443-3452.
[16] Polyzos, D. (2005), "3D frequency domain BEM for solving dipolar gradient elastic problems", Computational Mechanics 35, pp. 292-304.
[17] Polyzos, D., Tsepoura, K. G., Beskos, D. E. (2005), "Transient dynamic analysis of 3-D gradient elastic solids by BEM", Computers and Structures 83, pp. 783-792.
[18] Mindlin, R. D. (1964), "Microstructure in linear elasticity", Archive of Rational Mechanics and Analysis 16, pp. 51-78.
[19] Tsinopoulos, S. V., Kattis, S. E., Polyzos, D., Beskos, D. E. (1999), "An advanced boundary element method for axisymmetric elastodynamic analysis", Computer Methods in Applied Mechanics and Engineering 175, pp. 53-70.
[20] Mac Kenzie, J.K. (1950), "The elastic constants of solid containing spherical holes", *Proceedings of the Physical Society,* B63, 2-11**.**
[21] Foldy, L. L. (1945), "The multiple scattering of waves", Physical Review 67, pp. 107-119.