On the determination of the constants for a high-cycle model for sand

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ABSTRACT:
The paper discusses the determination of a set of constants for the high-cycle accumulation model (HCA) recently proposed by the authors (Niemunis et al., 2005). The HCA model may be used to predict permanent strains or excess pore water pressures in a non-cohesive soil due to a cyclic loading with relatively small amplitudes and a large number of cycles. The necessary laboratory tests and their analysis are explained in detail in this paper.

1 INTRODUCTION

The HCA model recently proposed by the authors (Niemunis et al., 2005) predicts permanent deformations or excess pore water pressures in non-cohesive soils due to many cycles ($N > 10^3$) with relatively small amplitudes ($\varepsilon^{\text{ampl}} < 10^{-3}$, so-called high- or polycyclic loading). The HCA model is based on an extensive laboratory testing program (Wichtmann, 2005) on a medium coarse sand. The model can be applied to foundations subjected to traffic loading, to offshore and onshore wind power plants, to machine foundations, to problems related to mechanical compaction of granular soils, etc.

In the HCA model a single material constant (critical friction angle φ_c) is used in the equations for the direction of accumulation $\mathbf{m} = \dot{\boldsymbol{\varepsilon}}^{\text{acc}}/\|\dot{\boldsymbol{\varepsilon}}^{\text{acc}}\|$. Eight material constants (C_e, C_p, C_Y, C_{N1}, C_{N2}, C_{N3}, $C_{\pi 1}$, $C_{\pi 2}$) are required for the intensity of accumulation $\dot{\varepsilon}^{\text{acc}} = \|\dot{\boldsymbol{\varepsilon}}^{\text{acc}}\|$. Furthermore, two material constants (e.g. Poisson's ratio ν, Young's modulus E) are needed for the isotropic elastic stiffness E coupling the predicted trends of stress and strain.

Although the test results and the constants for a medium coarse sand have already been presented by Wichtmann (2005) there is a need for a more detailed description of the procedure for the determination of the material constants. It is the objective of the present paper. The paper gives recommendations for laboratory tests and discusses the evaluation of the test results.

2 HIGH-CYCLE ACCUMULATION MODEL

The differences between the "implicit" and the "explicit" calculation strategy have been explained by Niemunis et al. (2005). The main constitutive equation of the HCA model for explicit calculations reads

$$\dot{\boldsymbol{\sigma}} = \mathsf{E} : (\dot{\boldsymbol{\varepsilon}} - \dot{\boldsymbol{\varepsilon}}^{\text{acc}} - \dot{\boldsymbol{\varepsilon}}^{\text{pl}}) \tag{1}$$

with the Jaumann stress rate $\dot{\boldsymbol{\sigma}}$ of the effective stress $\boldsymbol{\sigma}$, the strain rate $\dot{\boldsymbol{\varepsilon}}$, the prescribed strain accumulation rate $\dot{\boldsymbol{\varepsilon}}^{\text{acc}}$, the plastic strain rate $\dot{\boldsymbol{\varepsilon}}^{\text{pl}}$ and the (barotropic) elastic stiffness E. In the high-cyclic context "rate" means the derivative with respect to the number of cycles N, i.e. $\dot{\sqcup} = \partial \sqcup / \partial N$. The accumulation rate is calculated as the product of the scalar *intensity* of accumulation $\dot{\varepsilon}^{\text{acc}}$ and the *direction* of accumulation $\mathbf{m}$ (a unit tensor):

$$\dot{\boldsymbol{\varepsilon}}^{\text{acc}} \;=\; \dot{\varepsilon}^{\text{acc}}\,\mathbf{m} \;=\; f_{\text{ampl}}\;\dot{f}_N\;f_e\;f_p\;f_Y\;f_\pi\;\mathbf{m} \tag{2}$$

The flow rule of the modified Cam clay (MCC) model has been experimentally found to approximate $\mathbf{m}$ well. A multiplicative approach is used for the intensity of strain accumulation $\dot{\varepsilon}^{\text{acc}}$. Each function (see Table 1) considers separately the influence of a different parameter (strain amplitude $\varepsilon^{\text{ampl}}$, cyclic preloading, void ratio e, average mean pressure p^{av}, average stress ratio $\eta^{\text{av}} = q^{\text{av}}/p^{\text{av}}$ or $\bar{Y}$, polarization changes). In the function f_π, α is the angle between the current polarization and the so-called "back polarization". For the definition of a multiaxial amplitude and its polarization from a 6D strain path see (Niemunis et al., 2005).

3 DETERMINATION OF A SET OF CONSTANTS

The determination of the constants is demonstrated for a quartz sand with sub-angular grain shape. The grain size distribution curve is almost linear in the

Influencing parameter	Function	Material constants	Reference quantities
Strain amplitude	$f_{\text{ampl}} = \min\left\{\left(\varepsilon^{\text{ampl}}/\varepsilon^{\text{ampl}}_{\text{ref}}\right)^2; 100\right\}$		$\varepsilon^{\text{ampl}}_{\text{ref}} = 10^{-4}$
Cyclic preloading	$\dot{f}_N = \dot{f}_N^A + \dot{f}_N^B$	C_{N1}, C_{N2}, C_{N3}	
	$\dot{f}_N^A = C_{N1}C_{N2}\exp\left[-g^A/(C_{N1}f_{\text{ampl}})\right]$		
	$\dot{f}_N^B = C_{N1}C_{N3}$		
Void ratio	$f_e = \dfrac{(C_e - e)^2}{1+e}\dfrac{1+e_{\text{ref}}}{(C_e - e_{\text{ref}})^2}$	C_e	$e_{\text{ref}} = e_{\max}$
Average mean pressure	$f_p = \exp\left[-C_p\ (p^{\text{av}}/p_{\text{ref}} - 1)\right]$	C_p	$p_{\text{ref}} = p_{\text{atm}}$ = 100 kPa
Average stress ratio	$f_Y = \exp\left(C_Y\ \bar{Y}^{\text{av}}\right)$	C_Y	
Polarization changes	$f_\pi = 1 + C_{\pi 1}(1 - \cos\alpha)$	$C_{\pi 1}$	
	$\dot{\alpha} = -C_{\pi 2}\ \alpha\ (\varepsilon^{\text{ampl}})^2$	$C_{\pi 2}$	

Table 1. Summary of the functions, material constants and reference quantities of the high-cycle model

semi-logarithmic scale. The mean grain size is d_{50} = 0.6 mm and the coefficient of uniformity is C_u = 2.5.

3.1 *Index tests*

The maximum void ratio $e_{\max}$ is used as a reference quantity in the function f_e of the HCA model (Table 1). The minimum and maximum void ratios $e_{\min}$ and $e_{\max}$ may be determined according to the well-known procedures described in standard codes (e.g. DIN 18126 or ASTM-D-4253/ASTM-D-4254). For the present sand $e_{\max}$ = 0.856 and $e_{\min}$ = 0.495 have been obtained following the procedure of DIN 18126. For quartz sand the specific density may be assumed as ϱ_s = 2.65 g/cm^3.

3.2 *Tests on the critical friction angle φ_c*

The critical friction angle is needed in the equations for the "cyclic flow rule" **m**. φ_c may be obtained from the reposed angle. For this purpose a cone of sand is deposited by slow centric lifting of a funnel. φ_c is the inclination of the cone. For the present sand $\varphi_c = 33°$ was obtained as the mean value of five tests.

3.3 *Cyclic triaxial tests for C_e, C_p, C_Y, C_{N1}, C_{N2}, C_{N3}*

The constants of the functions f_e, f_p, f_Y and $\dot{f}_N$ may be obtained from load-controlled drained cyclic triaxial tests. The test device and the method of sample preparation used for the present study have been described by Wichtmann (2005).

In the following some remarks on suitable test conditions and test analysis are given:

- It is distinguished between the first "irregular" and the subsequent "regular" cycles (Fig. 1). The HCA model predicts the average accumulation during the regular cycles and the number of cycles N is counted from the beginning of the first regular cycle, i.e. $N = 1$ means after the first regular cycle.

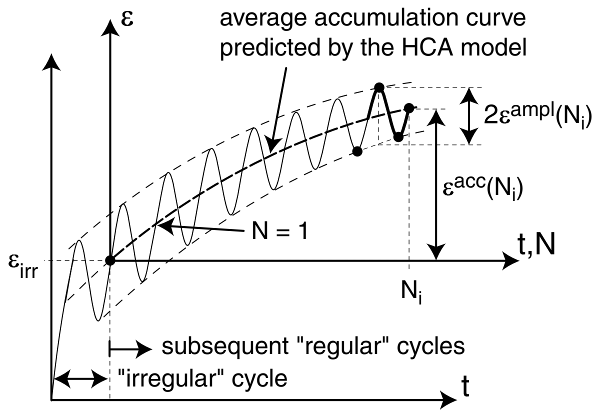

Figure 1. *Typical curve $\varepsilon(t)$ in a cyclic triaxial test*

- For sand the loading frequency does not significantly influence the residual and resilient deformations (Wichtmann, 2005) as long as inertia forces are negligible. For the determination of the constants of the HCA model, a loading frequency of 1 Hz may be suitable for most sands. In the case of fine sands with a low permeability a lower frequency might become necessary in order to assure drained conditions. Otherwise the pore water pressure u might significantly oscillate during a cycle or it might even accumulate with N. Due to the large deformations the first cycle in general should be applied slower (e.g. with a period T = 100 seconds) than the subsequent ones.

- The axial deformation and the volume change should be measured. It is sufficient to record the data at several time instances, e.g. from the irregular cycle, from the first 25 regular cycles and from 4 or 5 cycles at $N =$ 50, 100, 1,000, 2,000, 5,000, 10,000, ... Over a sampled cycle a sufficient number of data points (e.g. 50) should be recorded.

- The deformation during consolidation (a resting period of one hour at $\boldsymbol{\sigma}^{\mathrm{av}}$ has been chosen for the present tests) and during the irregular cycle is subtracted (Fig. 1), i.e. in the following Δh and ΔV mean the axial deformation and the volume change measured from the beginning of the first regular cycle. The axial and the volumetric strain are calculated from $\varepsilon_1 = \Delta h / h_0$ and $\varepsilon_v = \Delta V / V_0$ or $\varepsilon_1 = \ln(h_0/h)$ and $\varepsilon_v = \ln(V_0/V)$, respectively, with the initial height h_0, the initial volume V_0, the actual height $h = h_0 - \Delta h$ and the actual volume $V = V_0 - \Delta V$. "Initial" state means before the first regular cycle.

- The curves $\varepsilon_1(t)$ and $\varepsilon_v(t)$ during the regular cycles are splitted into an elastic portion ($\varepsilon_1^{\mathrm{ampl}}$, $\varepsilon_v^{\mathrm{ampl}}$) and a residual portion ($\varepsilon_1^{\mathrm{acc}}$, $\varepsilon_v^{\mathrm{acc}}$) (Fig. 1). The accumulated axial strain after the cycle $N = N_i$ may be calculated from $\varepsilon_1^{\mathrm{acc}} = \frac{1}{2}\left[\varepsilon_1^{\max}(N_{i+1}) + \varepsilon_1^{\min}(N_i)\right]$ wherein $\varepsilon_1^{\min}(N_i)$ e.g. means the minimum value of ε_1 during the cycle $N = N_i$. The elastic portion is $\varepsilon_1^{\mathrm{ampl}} = \frac{1}{2}\left[\varepsilon_1^{\max}(N_i) - \frac{1}{2}\left(\varepsilon_1^{\min}(N_i) + \varepsilon_1^{\min}(N_{i-1})\right)\right]$. The amplitude ($\varepsilon_v^{\mathrm{ampl}}$) and the residual value ($\varepsilon_v^{\mathrm{acc}}$) of the volumetric strain can be obtained in a similar manner.

- From ε_1 and ε_v the lateral strain is calculated from $\varepsilon_3 = \frac{1}{2}(\varepsilon_v - \varepsilon_1)$, the deviatoric strain from $\varepsilon_q = \frac{2}{3}(\varepsilon_1 - \varepsilon_3)$ and the total strain from $\varepsilon = \sqrt{(\varepsilon_1)^2 + 2\,(\varepsilon_3)^2}$. These equations can be directly applied to the amplitudes and the residual values, e.g. $\varepsilon^{\mathrm{ampl}} = \sqrt{(\varepsilon_1^{\mathrm{ampl}})^2 + 2(\varepsilon_3^{\mathrm{ampl}})^2}$ (don't use e.g. $\varepsilon^{\mathrm{ampl}} = \frac{1}{2}(\varepsilon^{\max} - \varepsilon^{\min})$).

- At least three test series are necessary: a series with different initial densities (Figure 2b), a series with a variation of the average mean pressure p^{av} (Figure 2c) and a series with different values of the average stress ratio η^{av} (Figure 2d). A test series with different stress amplitudes (Figure 2a) is useful in order to choose an appropriate amplitude ratio for the subsequent test series.

3.3.1 *Preliminary test series with different amplitudes*

This test series is recommended in order to choose an appropriate amplitude ratio for the subsequent test series. The tests may also be used in order to confirm

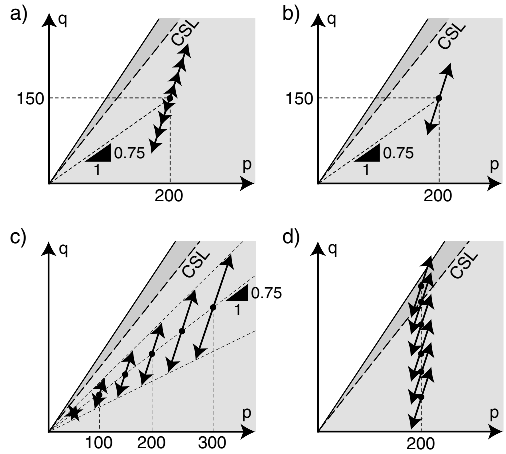

Figure 2. *Suitable stress paths in the cyclic triaxial tests for the determination of the constants C_e, C_p, C_Y, C_{N1}, C_{N2}, C_{N3} of the HCA model*

the function f_{ampl} of the HCA model. The tests should be performed with medium dense specimens (e.g. $I_{D0} = (e_{\max} - e)/(e_{\max} - e_{\min}) \approx 0.6$). We recommend to perform three tests in which at least $N = 10^5$ cycles are applied at a constant average stress. For the present study p^{av} = 200 kPa and $\eta^{\mathrm{av}} = 0.75$ (i.e. $K_0 = \sigma_3^{\mathrm{av}}/\sigma_1^{\mathrm{av}} = 0.5$) have been chosen. For uniform sands ($C_u \leq 3$) the amplitude ratio may be varied within the range $0.2 \leq \zeta = q^{\mathrm{ampl}}/p^{\mathrm{av}} \leq 0.4$. For sands with higher C_u-values lower values ($0.1 \leq \zeta \leq 0.3$) are recommended since the residual deformations increase with increasing C_u. In the present test series the chosen amplitude ratios ζ = 0.2, 0.3 and 0.4 correspond to stress amplitudes q^{ampl} = 40, 60 and 80 kPa. The regular cycles were applied with a loading frequency of 1 Hz.

The accumulated strain $\varepsilon^{\mathrm{acc}}$ is plotted versus the number of cycles N in Figure 3. The stress ratio for the subsequent test series on f_e, f_p and f_Y should be chosen large enough so that the residual deformations can be accurately measured. However, ζ should be chosen small enough in order to avoid excessive accumulation in tests on loose soil (f_e) or in tests with large stress ratios (f_Y). From the tests with different amplitudes, the amplitude ratio ζ of the test which leads to a moderate accumulation, say $1 \leq \varepsilon^{\mathrm{acc}} \leq 3\%$ after $N = 10^5$ cycles, could be chosen for the subsequent test series. According to Figure 3 the stress ratio ζ = 0.3 was used in the present study.

3.3.2 *Constant C_e of function f_e*

For the determination of C_e at least three tests with different initial densities of the specimens are advised (e.g. I_{D0} = 0.4, 0.6 and 0.8). All tests should be performed with the same average stress $\boldsymbol{\sigma}^{\mathrm{av}}$ and with the same stress amplitude. It is recommended to apply at least 10^5 cycles. If a test series with different stress amplitudes according to Section 3.3.1 has been con-

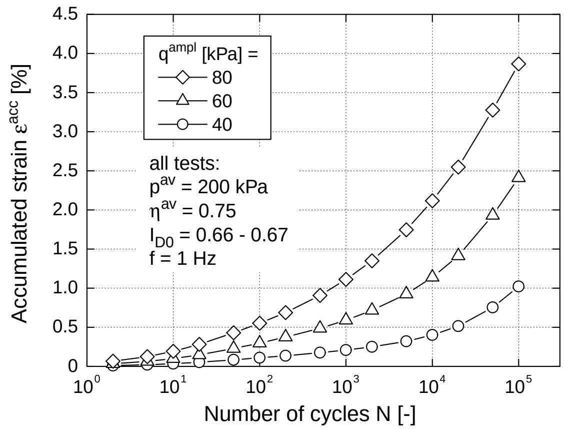

Figure 3. *Accumulation curves $\varepsilon^{acc}(N)$ in tests with different stress amplitudes q^{ampl}*

ducted the test with the chosen ζ-value and $I_{D0} \approx 0.6$ can be supplemented by two tests with a lower or a higher initial density, respectively. As an example, beside the test with I_{D0} = 0.66, p^{av} = 200 kPa, $\eta^{\text{av}} = 0.75$ and q^{ampl} = 60 kPa (Figure 3) two tests with I_{D0} = 0.52 and 0.82 have been performed. The accumulation curves of the three tests are given in Figure 4. Obviously the accumulation rate decreases with increasing I_{D0}.

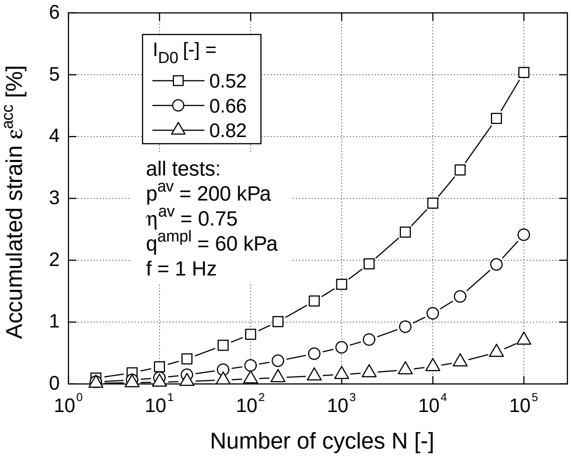

Figure 4. *Accumulation curves in tests with different initial relative densities I_{D0}*

For the determination of the constant C_e we need a presentation of the test results as given in Figure 5. It shows the accumulated strain after different numbers of cycles as a function of the void ratio. Since the tests are performed stress-controlled the strain amplitude $\varepsilon^{\text{ampl}}$ varies with N. Furthermore, the looser specimens have larger strain amplitudes. In order to "eliminate" the influence of the strain amplitude, the accumulated strain is divided by the amplitude function $\bar{f}_{\text{ampl}} = (\bar{\varepsilon}^{\text{ampl}}/\varepsilon_{\text{ref}}^{\text{ampl}})^2$ with $\varepsilon_{\text{ref}}^{\text{ampl}} = 10^{-4}$. For each number of cycles N a mean value of the strain amplitude $\bar{\varepsilon}^{\text{ampl}}$ has to be calculated. Since amplitude data is available at N-values in logarithmic distance (i.e. N = 1, 2, 5, 10 ...) the sum $\bar{\varepsilon}^{\text{ampl}} = 1/N \ \sum \varepsilon^{\text{ampl}}(N)$ is used, e.g. for $N = 10$: $\bar{\varepsilon}^{\text{ampl}} = 1/10 \ \{\varepsilon^{\text{ampl}}(N = 1) + \varepsilon^{\text{ampl}}(N = 2) + \frac{1}{2}[\varepsilon^{\text{ampl}}(N = 2) + \varepsilon^{\text{ampl}}(N = 5)] \cdot 3 + \frac{1}{2}[\varepsilon^{\text{ampl}}(N = 5) + \varepsilon^{\text{ampl}}(N = 10)] \cdot 5\}$. In a similar way also an integral value of the void ratio $\bar{e}$ is calculated for each N.

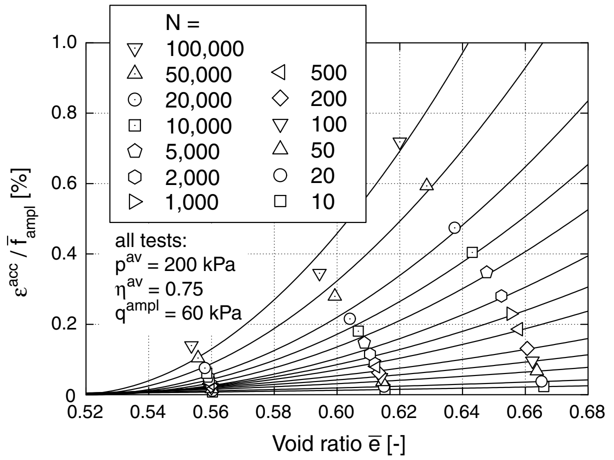

Figure 5. *Determination of constant C_e by fi tting of function f_e*

As an alternative the amplitudes or void ratios at lower numbers of cycles could receive a higher weighting by using $\bar{\varepsilon}^{\text{ampl}} = \frac{1}{i} \sum \varepsilon^{\text{ampl}}(N)$ where i is the number of $\varepsilon^{\text{ampl}}(N)$-values considered for the calculation, e.g. for $N = 10$: $\bar{\varepsilon}^{\text{ampl}} = \frac{1}{4}[\varepsilon^{\text{ampl}}(N = 1) + \varepsilon^{\text{ampl}}(N = 2) + \varepsilon^{\text{ampl}}(N = 5) + \varepsilon^{\text{ampl}}(N = 10)]$. Having $\bar{f}_{\text{ampl}}$ and $\bar{e}$ for a certain N, one can plot $\varepsilon^{\text{acc}}/\bar{f}_{\text{ampl}}$ versus $\bar{e}$ as it has been done in Figure 5.

For each N-value the function

$$f = k \ (C_e - e)^2/(1 + e) \tag{3}$$

has to be fitted to the data. The curve-fitting delivers the constant C_e. The constant k is not further used. The constants C_e obtained for the data in Figure 5 are summarized in Table 2. $N = 10$ was considered as the lowest N-value. A slight increase of C_e can be observed up to N = 200. For larger N-values, C_e is almost constant. The constant C_e may be simply taken as the mean value of the values determined for the different numbers of cycles. This results in C_e = 0.50. If the curve-fitting is restricted to a lower number of N-values (e.g. N = 20, 100, 10^3, 10^4 and 10^5) the same mean value C_e = 0.50 is obtained in this example. Thus, such a simplified procedure seems sufficient.

3.3.3 *Confirmation of f_{ampl}*

Having determined the constant C_e = 0.50, the function f_{ampl}, i.e. the linear proportionality between the accumulation rate $\dot{\varepsilon}^{\text{acc}}$ and the square of the strain amplitude $(\varepsilon^{\text{ampl}})^2$ may be confirmed. The data from the three tests with different amplitudes (Section 3.3.1) is used. As described in Section 3.3.2 integral values of void ratio $\bar{e}$ and strain amplitude $\bar{\varepsilon}^{\text{ampl}}$ are calculated

N	C_e	C_p	C_Y
10	0.43	0.42	1.4
20	0.45	0.41	1.6
50	0.47	0.38	1.9
100	0.48	0.36	2.1
200	0.50	0.33	2.3
500	0.51	0.33	2.5
1,000	0.51	0.36	2.7
2,000	0.52	0.40	2.8
5,000	0.52	0.43	2.8
10,000	0.52	0.47	2.8
20,000	0.52	0.49	2.9
50,000	0.52	0.49	2.9
100,000	0.52	0.51	2.7
Mean	0.50	0.41	2.4
Mean*	0.50	0.42	2.4

Table 2. Constants C_e, C_p, C_Y (* = mean value derived from the data at N = 20, 100, 10^3, 10^4 and 10^5 only)

for different N-values. Setting $\bar{e}$ into f_e (Table 1) delivers the quantity $\bar{f}_e$. For a given N the residual strain ε^{acc} has to be divided by the corresponding $\bar{f}_e$. In Figure 6, the values $\varepsilon^{\text{acc}}/\bar{f}_e$ are plotted versus the square of the strain amplitude $(\bar{\varepsilon}^{\text{ampl}})^2$. Straight lines through the origin confirm the function f_{ampl}.

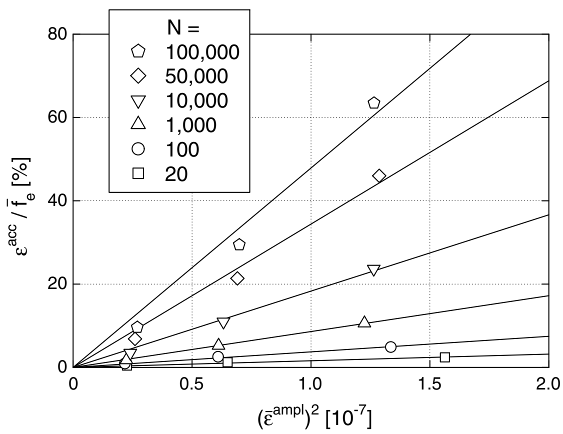

Figure 6. *Confirmation of function f_{ampl}*

3.3.4 Constant C_p of function f_p

For the determination of C_p tests with different average mean pressures p^{av} are necessary. It is recommended to supplement the test with $I_{D0} \approx 0.60$ and p^{av} = 200 kPa from the test series on the e-influence by additional tests with lower (e.g. p^{av} = 50 or 100 kPa) and higher (e.g. p^{av} = 300 kPa) pressures. The average stress ratio η^{av} and the amplitude ratio $\zeta = q^{\text{ampl}}/p^{\text{av}}$ should be kept constant within the test series. Figure 7 presents the accumulation curves $\varepsilon^{\text{acc}}(N)$ in four tests with η^{av} = 0.75 and ζ = 0.3. The accumulation rates are quite similar in the four tests. However, the increase of the strain amplitude with p^{av} due to the under-linear increase of the secant stiffness with p^{av} has to be considered.

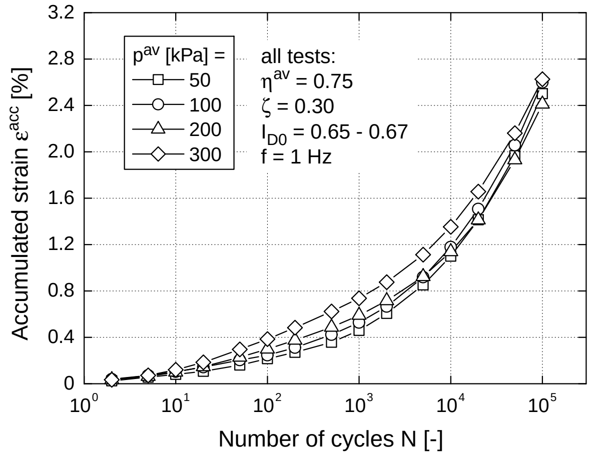

Figure 7. *Accumulation curves in tests with different average mean pressures p^{av}*

For each N the accumulated strain ε^{acc} is divided by the functions $\bar{f}_{\text{ampl}}$ and $\bar{f}_e$. They are calculated with $\bar{\varepsilon}^{\text{ampl}}$ and $\bar{e}$. The values $\varepsilon^{\text{acc}}/(\bar{f}_e\bar{f}_{\text{ampl}})$ are plotted versus p^{av} (Figure 8). The function

$$f = k\ \exp\left[-C_p\ \left(p^{\text{av}}/100 - 1\right)\right] \tag{4}$$

is fitted to the data resulting in the constant C_p. The C_p-values obtained from Figure 8 are summarized in Table 2. The mean value is C_p = 0.41. The simplified procedure delivers a similar constant C_p = 0.42.

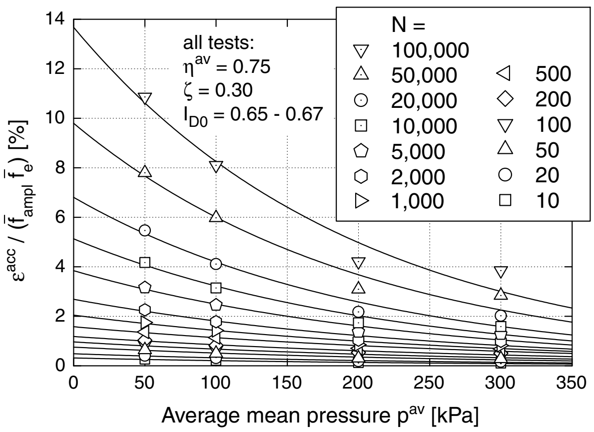

Figure 8. *Determination of constant C_p by fitting of function f_p*

3.3.5 Constant C_Y of function f_Y

The constant C_Y may be determined from tests with different average stress ratios η^{av} while keeping p^{av}, q^{ampl} and I_{D0} constant. The test with p^{av} = 200 kPa, $\eta^{\text{av}} = 0.75$ and $I_{D0} \approx 0.6$ from the series on C_e may be supplemented by tests with lower (e.g. η^{av} = 0.5) and higher (e.g. η^{av} = 1.0 or 1.25) average stress ratios. Figure 9 presents the accumulation curves in tests

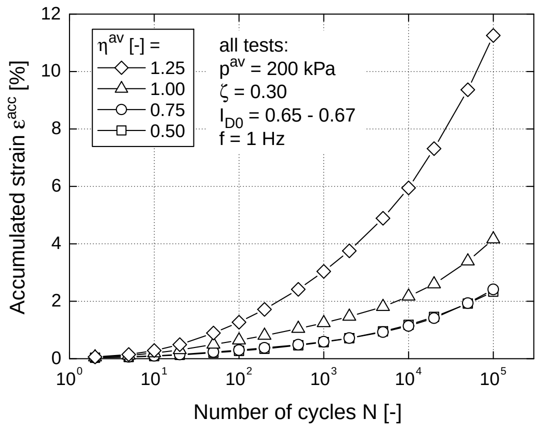

Figure 9. *Accumulation curves in tests with different average stress ratios η^{av}*

with η^{av} = 0.5, 0.75, 1.0 and 1.25. The accumulation rate increases with increasing average stress ratio.

As a measure of the stress ratio we use $\bar{Y} = (Y - 9)/(Y_c - 9)$ with $Y = 27(3+\eta)/[(3+2\eta)(3-\eta)]$ and $Y_c = (9 - \sin^2\varphi_c)/(1 - \sin^2\varphi_c)$ with the critical friction angle φ_c. In order to determine C_Y the residual strain after different N-values is divided by $\bar{f}_e$ and $\bar{f}_{\text{ampl}}$ (since the strain amplitude $\varepsilon^{\text{ampl}}$ decreases with increasing η^{av}) and plotted versus the stress ratio $\bar{Y}^{\text{av}}$ of the test (Figure 10). The function

$$f = k\ \exp(C_Y \bar{Y}^{\text{av}}) \tag{5}$$

is fitted to the data. The obtained constants C_Y are summarized in Table 2. An increase of C_Y during the first 500 cycles is obvious while the values stay almost constant at larger N. A mean value C_Y = 2.4 has been determined.

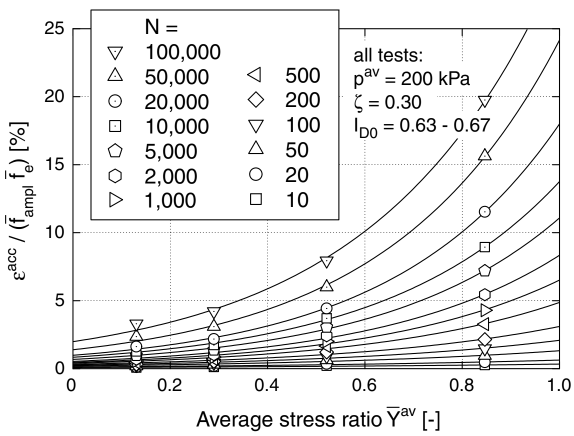

Figure 10. *Determination of constant C_Y by fi tting of function f_Y*

3.3.6 *Constants C_{N1}, C_{N2} and C_{N3} of function $\dot{f}_N$*

Having determined C_e, C_p and C_Y the constants C_{N1}, C_{N2} and C_{N3} of function $\dot{f}_N$ can be obtained. The accumulation curves $\varepsilon^{\text{acc}}(N)$ of all tests described in Sections 3.3.1 to 3.3.5 are used for this purpose. The residual strain ε^{acc} is divided by the functions $\bar{f}_{\text{ampl}}$, $\bar{f}_e$, f_p and f_Y and plotted versus N. The functions are calculated with the integral values $\bar{\varepsilon}^{\text{ampl}}$ and $\bar{e}$ and with the p^{av}- and the $\bar{Y}^{\text{av}}$-values of the respective tests. The data in Figure 11 was obtained using C_e = 0.50, C_p = 0.42 and C_Y = 2.4 as determined from the simplified procedure (i.e. with only some of the N-values, Table 2). The function

$$f_N \quad = \quad C_{N1}[\ln(1 + C_{N2}N) + C_{N3}N] \tag{6}$$

has to be fitted to all data points in Figure 11. The constants $C_{N1} = 2.4 \cdot 10^{-3}$, $C_{N2} = 0.026$ and $C_{N3} = 8.2 \cdot 10^{-5}$ have been obtained for the present sand. The fitted function is given as a solid line in Figure 11.

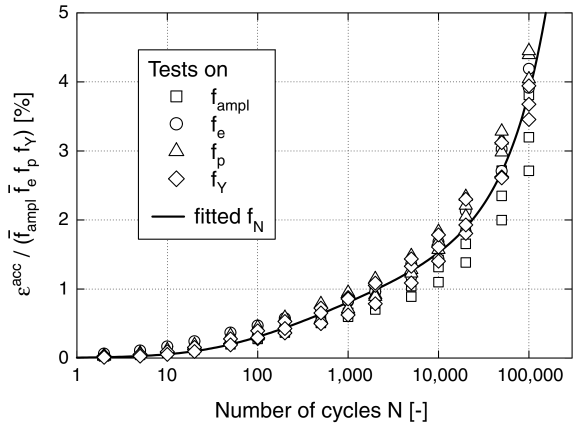

Figure 11. *Fitting of function f_N to the data of all test series*

If the boundary value problems calculated with the HCA model involve very large numbers of cycles $N \geq 10^6$ a long-term test should be performed and the function f_N should be fitted to the data of this test.

3.4 *Constants $C_{\pi1}$ and $C_{\pi2}$ of function f_π*

BVPs with a constant polarization of the cycles may be calculated using $f_\pi = 1$ as a first approximation. If the polarization varies, the constants $C_{\pi1}$ and $C_{\pi2}$ of function f_π have to be determined. Tests with a sudden change of the direction of the cycles are necessary. A suitable test is the cyclic multiaxial simple shear test ((Wichtmann, 2005)).

The increase of the accumulation rate after a change of the polarization is obvious in Figure 12 which presents accumulation curves $\varepsilon^{\text{acc}}(N)$ in tests with and without a sudden $90°$-change of the polarization. In Figure 13 the ratio of the accumulation rates $\dot{\varepsilon}^{\text{acc}}$ in the tests with and without a change of the polarization (= factor f_π) is plotted versus the number of cycles $N - N_{cp}$ after the change of polarization. f_π decays during approx. 1,000 cycles. The curve $f_\pi(N - N_{cp})$ seems to be independent of the initial density. Thus, a pair of tests with and without a change of the polarization is sufficient in order to determine $C_{\pi1}$ and $C_{\pi2}$.

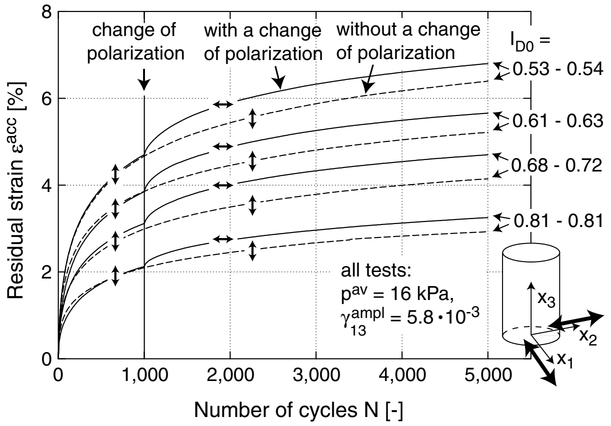

Figure 12. *Temporary increase of the accumulation rate due to a sudden* $90°$*-change of the polarization, results of cyclic multiaxial simple shear tests*

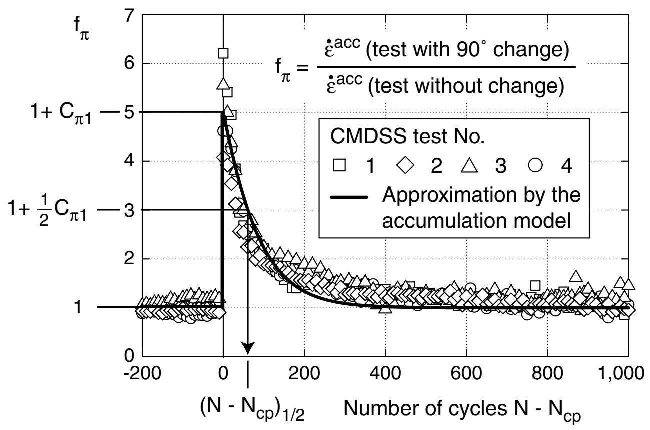

Figure 13. *Decay of the increased accumulation rate during the cycles following the* $90°$*-change of the polarization*

If the polarization is not changed $\cos\alpha = 1$ and $f_\pi = 1$ hold. Directly after a change of the polarization by $90°$, $\cos\alpha = 0$ and $f_\pi = 1 + C_{\pi 1}$ are valid. The material constant $C_{\pi 1}$ is the difference between f_π directly before and f_π directly after the polarization change. From the four tests $C_{\pi 1} = 4.0$ was determined as a mean value.

The rate of decay is governed by the second material constant $C_{\pi 2}$. It can be determined from $C_{\pi 2} = \dfrac{\ln(3/2)}{(\varepsilon^{\text{ampl}})^2\,(N - N_{\text{cp}})_{1/2}}$ with $(N - N_{\text{cp}})_{1/2}$ being the number of cycles for which the factor f_π takes the value $1 + C_{\pi 1}/2$ (Figure 13). The number of cycles $(N - N_{\text{cp}})_{1/2}$ can be seen as a kind of "half-life" of the polarization effect. For the four tests in Figure 13, $C_{\pi 2} = 200$ has been obtained.

A disadvantage of the simple shear test is the inhomogeneous field of stress and strain within the specimen. As an alternative a cyclic triaxial test with a sudden $90°$ change of the polarization of 1-d cycles in the P-Q-plane could be performed. $P = \sqrt{3}p$ and $Q = \sqrt{2/3}q$ are the isomorphic stress invariants.

3.5 *Constants of elasticity* E

For E in Equation (1) an isotropic (hypo)elastic stiffness with a barotropic Young's modulus is set into approach. E can be developed by comparing the rate of cyclic relaxation and the rate of cyclic creep under the same conditions ($\boldsymbol{\sigma}^{\text{av}}$, e, N) and under the same cyclic load. Two elastic constants (e.g. Poisson's ratio ν and bulk modulus $K(p)$) have to be determined. Poisson's ratio may be estimated as ν = 0.2 to 0.3. The bulk modulus $K = \dot{u}/\dot{\varepsilon}_v^{\text{acc}}$ can be obtained from comparing the rate of pore water pressure $\dot{u}$ in an undrained cyclic test and the rate of volumetric strain accumulation $\dot{\varepsilon}_v^{\text{acc}}$ in a drained cyclic test. A study of K with 15 pairs of drained and undrained cyclic triaxial tests on a medium coarse sand has been documented by Wichtmann et al. (2007). For the first approximation it is recommended to use $K = A\ p_{\text{atm}}^{1-n}\ p^n$ with A = 540 and n = 0.30.

4 SUMMARY AND CONCLUSIONS

The paper presents a procedure for the determination of a set of material constants for the HCA model proposed by Niemunis et al. (2005). The critical friction angle φ_c is determined as the angle of repose. The constants C_e, C_p, C_Y, C_{N1}, C_{N2} and C_{N3} may be determined from at least nine drained stress-controlled cyclic triaxial tests. The constants $C_{\pi 1}$ and $C_{\pi 2}$ can be obtained from two cyclic (multiaxial simple shear or triaxial) tests, one with and one without a sudden change of the direction of the cycles. The constants of the elastic stiffness E may be determined by comparing drained and undrained cyclic tests. The paper discusses suitable test conditions and the analysis of the test results.

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